

Planck-scale nonthermal correlations in a noncommutative geometry inspired Vaidya black hole

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Abstract

Using the noncommutative geometry inspired Vaidya metric obtained in terms of coordinate coherent states and also utilizing the generalized uncertainty principle (GUP), we show that the nonthermal nature of the Hawking spectrum leads to Planck-scale nonthermal correlations between emitted modes of evaporation. Our analysis thus exhibits that owing to self-gravitational effects plus noncommutativity and GUP influences, information can emerge in the form of Planck-scale correlated emissions from the black hole.

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1 Introduction

The discovery of black hole evaporation from the Hawking proposal [1] has led to a long-lived discussion dealing with the so-called *black hole information paradox* (for reviews see [2]) which implies the purely thermal essence of the spectrum. It is widely believed that a reply to the information loss problem could provide a key element in the quest for a yet to be formulated theory of quantum gravity. In quantum mechanics, information is preserved by unitarity. While, in black hole physics, the information conveyed by a physical procedure descends to the singularity, and therefore has never been retrieved. This would allow a nonunitary quantum evolution that maps a pure state to a mixed state [3]. The reply to the query of whether or not this contravention transpires will profoundly describe the process in which quantum mechanics and general relativity (e.g., black hole physics) would appear as borders of the quantum gravity hypothesis.

There are many conjectures to preserve the fundamental principles of quantum mechanics near black holes [2]. A common conjecture is that information actually comes out of the black hole as nonthermal correlations between different modes of radiation during evaporation. The Hawking effect can be described as the quantum tunneling of vacuum fluctuations across the horizon. This is achieved either by the radial null geodesic approach [4] or by the Hamilton-Jacobi approach [5] to compute the tunneling probability. In these methods, the credibility of ordinary perspective for the radiation procedure will fail owing to the fact that, in the final phase of black hole evaporation, the effects of the gravitational back-reaction should be taken into consideration. In Parikh-Wilczek tunneling [4], one takes into account the back-reaction consequences in a definite gap between the initial and final radii as a result of self-gravitation effects of outgoing shells, which is the classically forbidden path, (i.e., the barrier). From the other point of view, the tunneling barrier, which guarantees energy conservation throughout the evaporation is created by a reduction in the black hole horizon just by the emitted particle itself which leads to nonthermal corrections to the black hole radiation spectrum. However, the authors of Refs. [6, 7] (incorrectly) show that the nonthermal spectrum by itself does not solve the information loss problem and the form of the corrections, due to the absence of correlations between the tunneling probabilities of different modes in the black hole radiation spectrum, is not sufficient by itself to regain the information. This incorrect claim was corrected by Zhang *et al.* [8] (see also [9]). In 2009, Zhang *et al.*, by using a standard statistical method and based on the results within a semiclassical treatment for

s-wave emissions, discovered the existence of correlations within Hawking radiation from a black hole. They demonstrated that black hole radiation as tunneling is an entropy conservation process, and information leaks out via radiation, which clearly leads to the conclusion that the process of Hawking radiation is unitary and therefore no information loss appears. The mistake in [6, 7] was a statistical error driven from statistically independent events. When performing the method of [6, 7] in a purely thermal spectrum, correlations are observable, which is patently untrue. Whereas in the method of [8] no correlations appear for a purely thermal spectrum.

Lately, a new interesting model of noncommutativity in terms of coherent states is proposed [10], which guarantees Lorentz invariance, unitarity and UV-finiteness of quantum field theory. The authors in [11] used this method to establish a physically inspired type of noncommutativity corrections to black hole solutions (coordinate coherent state (CCS) approach). In this model, the point-like structure of mass M , in lieu of being completely localized at a point, is portrayed by a smeared structure throughout a region of linear size $\sqrt{\theta}$ (see also [12]). Using the CCS approach, it has been exhibited that the modified metric does not allow the black hole to decay below the Planckian relic. The evaporation process terminates when the size of the black hole reaches a Planck size remnant, interpreting a black hole released of curvature singularity in the origin. Because spacetime noncommutativity can cure some kinds of divergences that appear in general relativity, we hope to make some improvements in evaporation process computations and generalize the tunneling picture using the CCS method. In 2011, Zhang *et al.* accomplished some work in this direction [13]. They discovered correlations that can carry information about noncommutativity in Hawking radiation from noncommutative black holes.

Besides, it is by now widely accepted that measurements in quantum gravity should be determined by the generalized uncertainty principle (GUP) [14]. In other words, the so-called Heisenberg uncertainty principle (HUP), should be re-formulated owing to the noncommutative nature of spacetime at the Planck scale. As a result, it has been pointed out that in quantum gravity there exists a minimal observable distance of the order of the Planck length, which is an immediate consequence of the GUP. Because quantum gravity proposals prevalently anticipate the existence of a minimal observable length of the order of the Planck length [14], the application of the GUP to black hole thermodynamics has attracted considerable attention and leads to significant modifications to the emission process, particularly at the final stages of evaporation (many authors considered various problems in this framework, e.g. see [15]). Recently, we have modified the Parikh-Wilczek

tunneling methodology by including quantum gravity effects that were revealed in the existence of a minimal observable length [16, 17, 18, 19]. Indeed, the self-gravitation influences with inclusion of Planck-scale modification cannot be neglected, particularly once the black hole mass becomes comparable to the Planck mass.

In the study of black hole evaporation, there has been a significant point raised concerning how black hole mass reduces as a back-reaction of the Hawking radiation. Because the dynamics for the mass of an evaporating black hole is a persistent problem, we apply the noncommutative geometry inspired Vaidya metric derived in Refs. [20, 21] to find the Planck-scale nonthermal correlations within the Hawking radiation. We investigate the tunneling methodology by the radial null geodesic approach in the background of CCS noncommutativity including Planck-scale corrections from the GUP origin. When the effects of gravitational back-reaction including CCS noncommutativity are incorporated with the Planck-scale corrections via the GUP, one would recognize the occurrence of Planck-scale correlations between the tunneling probability of different modes in the black hole radiation spectrum. The appearance of these correlations can shed more light on the information loss problem.

The paper is organized as follows. In Sec. 2, using the influences of noncommutativity in the context of CCS for the Vaidya metric, we take into account the GUP effects, an achievable role of quantum gravity, in the Parikh-Wilczek tunneling method. The tunneling amplitude at which massless particles tunnel across the event horizon is calculated and the results, a nonthermal spectrum for the escape of information via the Hawking radiation, is exhibited, namely the appearance of Planck-scale nonthermal correlations. Finally, a summary is presented in Sec. 3.

2 Noncommutativity and GUP influences on a Vaidya black hole

According to [11, 12], the simple idea of point-like particles turns into a physically irrelevant concept and should be replaced by a gaussian mass or energy distribution with a minimal width that corresponds to the principles of quantum mechanics. The procedure we use here is to seek for a nonstatic, spherically symmetric, asymptotically flat structure with a minimal width and gaussian distribution of mass or energy, whose noncommutative size is characterized by the parameter $\sqrt{\theta}$. To this purpose, the mass or energy

distribution can be written as

$$\rho_\theta = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}}, \quad (1)$$

where, ρ_θ and M are functions of both t and r . Because a nonstatic and spherically symmetric spacetime is contingent upon an arbitrary dynamical mass function, it may be suitably exhibited by the Vaidya solution [22, 23]. This kind of black hole is considered the illustration of a more practical one due to its time-dependent decreasing mass on account of the evaporation procedure. In this paper, we use the diagonal form of the Vaidya metric with respect to $\{x^\mu\} = \{t, r, \vartheta, \phi\}$ coordinates, ($\mu = 0, 1, 2, 3$), as given by Farley and D'Eath [24][†]

$$ds^2 = -e^{b(t,r)} dt^2 + e^{a(t,r)} dr^2 + r^2 d\Omega^2, \quad (2)$$

where $d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\phi^2$. The Einstein field equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$, for a spherically symmetric geometry of (2) lead to the following relations[‡]:

$$a' - \frac{1 - e^a}{r} = 8\pi r T_{rr}, \quad (3)$$

$$b' + \frac{1 - e^a}{r} = 8\pi r e^{a-b} T_{tt}, \quad (4)$$

$$T_{rr} = e^{a-b} T_{tt}, \quad (5)$$

$$\dot{a} = 8\pi r T_{tr}, \quad (6)$$

where the prime abbreviates $\partial/\partial r$, and the overdot abbreviates $\partial/\partial t$. Using these equations, one can find

$$\frac{a' - b'}{2} = \frac{1 - e^a}{r}. \quad (7)$$

Now, corresponding to a spherically symmetric null-fluid source, we can conclude the following expressions for e^{-a} and e^b :

$$e^{-a(t,r)} = 1 - \frac{2M(t,r)}{r}, \quad (8)$$

$$e^{b(t,r)} = \left(\frac{\dot{M}}{\chi(M)} \right)^2 e^{-a}, \quad (9)$$

[†]See also [25] for a more detailed study of the semiclassical methods which lead to the Hawking temperature in the Vaidya black hole.

[‡]We use the natural units, i.e. $\hbar = c = G = 1$.

where $\chi(M) = M'(1 - 2M/r)$ is the arbitrary positive function of t and r . We now relate the diagonal form of the Vaidya metric, as given in Eq. (2), to the null form of the metric. The Vaidya null dust collapse model is described by a spherically symmetric, nonstatic spacetime with a metric in terms of null coordinates (u, r, ϑ, ϕ) . The Vaidya null dust collapse is extensively studied in the literature [26]. In 2001, Claudel, Virbhadra, and Ellis [27] have done some very important work in this area. They presented a comprehensive paper about the geometry of photon surfaces and proved some important theorems on photon sphere. They have shown that the naked central singularity for a Vaidya null dust collapse is enclosed within the photon surface in the sense that any partial Cauchy surface extending to spatial infinity must intersect the photon surface in a two-sphere (to perceive implications of photon spheres for astrophysics, see [28, 29, 30]).

In the (u, r, ϑ, ϕ) coordinate system, the Vaidya null dust collapse model has the following form

$$ds^2 = - \left(1 - \frac{2M(u)}{r} \right) du^2 + 2dudr + r^2 d\Omega^2. \quad (10)$$

It is clear that (10) is of the Eddington-Finkelstein type [31]. The radially outgoing null geodesics are exactly paths of constant u . The function M is now independent of r and constant along outgoing null rays. In the general situation that dM/du is not known, it has demonstrated unfeasible to diagonalise the Vaidya metric and to determine u as an explicit function of t and r . In view of the fact that $\dot{M} < 0$ and $M' > 0$, one obtains that, along lines: $u = \text{constant}$, r increases with increasing t . An alteration of variables: $(u, r) \rightarrow (t, r)$, which agrees asymptotically with the requirement $u = (t - r)$, can be found by the coordinate transformation [24]

$$du = - \left(\frac{\dot{M}}{\chi(M)} \right) dt - \left(\frac{M'}{\chi(M)} \right) dr. \quad (11)$$

To have an exact $(t - r)$ dependent case of the metric, we present a Schwarzschild-like metric for the Vaidya solution instead of a standard Eddington-Finkelstein metric. In the following, to make the problem well-behaved, we choose $\chi(M) = -\dot{M}$. This metric resembles the Schwarzschild spacetime, excluding that the role of the Schwarzschild mass is performed by a mass function $M(t, r)$, which changes extremely gradually with respect to both t and r in the spacetime region including the outgoing radiation [32]. Hence, the corresponding geometry in this area containing the radially outgoing radiation is of a slowly varying Vaidya type, that is, $\dot{M} \ll 1$ and $M' \ll 1$.

According to [20, 21], the noncommutative geometry inspired Vaidya metric in the presence of a smeared mass or energy source, by solving Einstein equations with (1) as a matter source, can be found as

$$ds^2 = -F(t, r)dt^2 + F^{-1}(t, r)dr^2 + r^2d\Omega^2, \quad (12)$$

with

$$F(t, r) = 1 - \frac{2M_\theta(t, r)}{r}, \quad (13)$$

where the gaussian-smeared mass distribution is

$$M_\theta(t, r) = M_I \left[\mathcal{E} \left(\frac{r-t}{2\sqrt{\theta}} \right) \left(1 + \frac{t^2}{2\theta} \right) - \frac{r}{\sqrt{\pi\theta}} e^{-\frac{(r-t)^2}{4\theta}} \left(1 + \frac{t}{r} \right) \right], \quad (14)$$

where M_I is the initial black hole mass and $\mathcal{E}(x)$ displays the *Gauss error function* specified as $\mathcal{E}(x) \equiv 2/\sqrt{\pi} \int_0^x e^{-p^2} dp$. Line element (12) portrays the geometry of a noncommutative inspired Vaidya black hole. It is obvious that metric (12) has a coordinate singularity at the event horizon as

$$r_H = 2M_\theta(t, r_H). \quad (15)$$

Note that because there is no analytical solution for r_H versus M_I , one can approximately compute the noncommutative horizon radius versus the initial mass by setting $r_H = 2M_I$ into the function of gaussian-smeared mass distribution $M_\theta(t, r_H)$, namely

$$r_H = 2M_\theta(t, M_I) = 2M_I \left[\mathcal{E} \left(\frac{2M_I - t}{2\sqrt{\theta}} \right) \left(1 + \frac{t^2}{2\theta} \right) - \frac{2M_I}{\sqrt{\pi\theta}} e^{-\frac{(2M_I - t)^2}{4\theta}} \left(1 + \frac{t}{2M_I} \right) \right]. \quad (16)$$

The radiating property of such a modified vaidya black hole can now be inspected by the quantum tunneling procedure proposed in Ref. [4]. To describe the quantum tunneling approach wherein a particle travels in a dynamic geometry and crosses the horizon without singularity on the path, we should use a coordinate system that is not singular at the horizon. Painlevé coordinates [33] which are utilized to remove coordinate singularity are specifically appropriate choices in this method. Under the Painlevé time coordinate transformation, we have

$$dt \rightarrow dt - \frac{\sqrt{1 - F(t, r)}}{F(t, r)} dr, \quad (17)$$

the noncommutative Painlevé metric now immediately reads

$$ds^2 = -F(t, r)dt^2 + 2\sqrt{1 - F(t, r)}dt dr + dr^2 + r^2d\Omega^2$$

$$= - \left(1 - \frac{2M_\theta(t, r)}{r} \right) dt^2 + 2\sqrt{\frac{2M_\theta(t, r)}{r}} dt dr + dr^2 + r^2 d\Omega^2. \quad (18)$$

This metric is stationary, and there exists no coordinate singularity at the horizon. The outgoing radial null geodesics are given by

$$\dot{r} = 1 - \sqrt{1 - F(t, r)} = 1 - \sqrt{\frac{2M_\theta(t, r)}{r}}, \quad (19)$$

where $\dot{r} \equiv dr/dt$. In accordance with the original work by Parikh and Wilczek [4], the WKB approximation is valid at the neighborhood of the horizon. Therefore, the tunneling probability for the classically prohibited area as a function of the imaginary part of the action for a particle in a tunneling procedure takes the form [§]

$$\Gamma \sim e^{-2\text{Im } I}. \quad (20)$$

Here, we consider a spherical positive energy shell including the ingredients of massless particles each of which moves on a radial null geodesic like an s -wave outgoing particle that passes through the horizon in the outward direction from r_{in} to r_{out} . So, the imaginary part of the action is given by

$$\text{Im } I = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr. \quad (21)$$

If we consider the particle's self-gravitation effect, in conformity with the original work suggested by Kraus and Wilczek [38], then both the noncommutative Painlevé metric and the geodesic equation should be modified by the response of the background geometry.

As indicated briefly in the introduction, a consequential anticipation of various scenarios of quantum gravity is the existence of a minimal observable distance on the order of the Planck length that cannot be probed [14], for example, in string theory there exists a constraint on probing distances smaller than the string length. Therefore the HUP is modified to incorporate this constrained resolution of spacetime points. The consequence of this modification is the so-called GUP, which in fact has its origin in the quantum fluctuations of the spacetime at the Planck scale. The form of the GUP in terms of the Planck length, L_{Pl} , can be represented as follows:

$$\Delta x \geq \frac{1}{\Delta p} + \alpha L_{Pl}^2 \Delta p, \quad (22)$$

[§]Note that there is another standpoint on using Eq. (20); there exists a problem here recognized as *the factor 2 problem* [34, 35, 36]. In Ref. [37], a solution to this problem was prepared concerning the overlooked temporal contribution to the tunneling amplitude.

where α is a dimensionless constant on the order of one that depends on the details of quantum gravity theory. In the limit $\Delta x \gg L_{Pl}$, the HUP is recovered (i.e., $\Delta x \Delta p \geq 1$). The second term on the right-hand side of the GUP relation plays an essential role when the momentum and distance scales are in the vicinity of the Planck scale. In an innovative method, by applying the HUP, the thermodynamical quantities for a spherical black hole can be achieved [39]. Also, the application of the GUP to black hole thermodynamics in the same method modifies the results by inclusion of quantum gravity influences on the ultimate phases of the evaporation process with an abundant phenomenology [15].

Here, we apply the GUP to find the Planck-scale information in the black hole evaporation procedure. In this setup, we use the method appearing in Ref. [8] to recover information from the Hawking radiation. We are going to investigate the modifications of the Hawking radiation via the tunneling process by using the GUP-corrected de Broglie wavelength, the squeezing of the fundamental momentum-space cell (see for instance [40] and references therein), and then a GUP-corrected energy

$$\lambda \simeq \frac{1}{p} \left(1 + \alpha L_{Pl}^2 p^2 \right), \quad (23)$$

$$E \simeq E(1 + \alpha L_{Pl}^2 E^2). \quad (24)$$

Now, in the tunneling process, it is necessary to take into account the reaction of the background geometry with an emitted GUP-corrected energy E . We hold the total ADM mass (M_I) of the spacetime fixed, and allow the hole mass to fluctuate. In other words, a massless particle as a shell travels on the geodesics of a spacetime with M_I replaced by $M_I - E$. Next, we should first substitute $M_I - E$ for M_I in Eq. (19) and then apply the deformed Hamilton's equation of motion [16, 17, 18],

$$\dot{r} \simeq \left(1 + \alpha L_{Pl}^2 E^2 \right) \frac{dH}{dp_r} \Big|_r, \quad (25)$$

to alter the integral variable of the imaginary action (21) from momentum to energy. So, we have

$$\text{Im } I = \text{Im} \int_{r_{in}}^{r_{out}} \int_{M_I}^{M_I - E} \frac{1 + \alpha L_{Pl}^2 E'^2}{\dot{r}} dH dr, \quad (26)$$

where the hamiltonian is $H = M_I - E'$. We evaluate integral (26) by writing the explicit form for the radial null geodesic, which includes back-reaction effects, namely

$$\dot{r} = 1 - \sqrt{\frac{2M_\theta(t, M_I - E)}{r}}, \quad (27)$$

where

$$M_\theta(t, M_I - E) = (M_I - E) \left[\mathcal{E} \left(\frac{2(M_I - E) - t}{2\sqrt{\theta}} \right) \left(1 + \frac{t^2}{2\theta} \right) - \frac{2(M_I - E)}{\sqrt{\pi\theta}} e^{-\frac{(2(M_I - E) - t)^2}{4\theta}} \left(1 + \frac{t}{2(M_I - E)} \right) \right]. \quad (28)$$

Thus, we find

$$\text{Im } I = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^E \frac{1 + \alpha L_{Pl}^2 E'^2}{1 - \sqrt{\frac{2M_\theta(t, M_I - E')}{r}}} (-dE') dr. \quad (29)$$

The r integral of Eq. (29) can be done first by deforming the contour for the lower half E' plane. Finally, the imaginary part of the action yields the following form:

$$\text{Im } I = \text{Im} \int_0^E 4\pi i M_\theta(t, M_I - E') \left(1 + \alpha L_{Pl}^2 E'^2 \right) dE', \quad (30)$$

which can be achieved as

$$\begin{aligned} \text{Im } I = & 3\pi\theta(\mathcal{E}_2 - \mathcal{E}_1) + \sqrt{\pi\theta}e^{-u}[(6M_I + 5t)(e^v - e^w) + 6Ee^w] + \pi\alpha L_{Pl}^2 \left[(\mathcal{E}_2 - \mathcal{E}_1) \right. \\ & \times \left(\frac{15}{16}t^4 - \frac{10}{3}M_I t^3 + 3M_I^2 t^2 - \frac{M_I^4}{3} \right) + \mathcal{E}_2 E^3 \left(\frac{16}{3}M_I - 5E \right) \Big] + \pi \left[(\mathcal{E}_2 - \mathcal{E}_1) \right. \\ & \times \left(3t^2 - 2M_I^2 \right) + 2\mathcal{E}_2 E(2M_I - E) \Big] + \sqrt{\frac{\pi}{\theta}} t^2 e^{-u} \left[\alpha L_{Pl}^2 \left[e^v \left(\frac{t^3}{16} + \frac{M_I t^2}{24} - \frac{M_I t}{8} \right. \right. \right. \\ & \left. \left. - \frac{M_I^2}{12} \right) - e^w \left[\frac{t^3}{16} - t^2 \left(\frac{5M_I}{24} + \frac{E}{8} \right) + t \left(\frac{M_I E}{6} + \frac{M_I^2}{12} + \frac{E^2}{4} \right) + \frac{E M_I^2}{6} - \frac{3E^3}{2} \right. \right. \right. \\ & \left. \left. \left. + \frac{M_I^3}{6} + \frac{M_I E^2}{6} \right] \right] + \left(M_I + \frac{t}{2} \right) (e^v - e^w) + E e^w \right] + \frac{\pi}{\theta} \left[\alpha L_{Pl}^2 \left[(\mathcal{E}_2 - \mathcal{E}_1) \left(\frac{t^6}{32} - \frac{M_I t^5}{6} \right. \right. \right. \\ & \left. \left. \left. + \frac{M_I^2 t^4}{4} - \frac{M_I^4 t^2}{6} \right) + t^2 E^3 \mathcal{E}_2 \left(\frac{8M_I}{3} - \frac{5E}{2} \right) \right] + (\mathcal{E}_2 - \mathcal{E}_1) \left(\frac{t^4}{4} - M_I^2 t^2 \right) \right. \\ & \left. \left. + \mathcal{E}_2 t^2 E(2M_I - E) \right] + O(\alpha^2 L_{Pl}^4), \quad (31) \end{aligned}$$

where

$$\begin{cases} \mathcal{E}_1 \equiv \mathcal{E} \left(\frac{2M_I - t}{2\sqrt{\theta}} \right) \\ \mathcal{E}_2 \equiv \mathcal{E} \left(\frac{2M_I - 2E - t - 2\alpha L_{Pl}^2 E^3}{2\sqrt{\theta}} \right) \\ u \equiv \frac{M_I^2 + E^2 + Et + \alpha L_{Pl}^2 (2E^4 + E^3 t)}{\theta} \\ v \equiv u - \frac{t^2 + 4M_I(M_I - t)}{4\theta} \\ w \equiv \frac{4M_I(2E + t + 2\alpha L_{Pl}^2 E^3) - t^2}{4\theta}. \end{cases}$$

We consider the leading-order correction to be just proportional to (αL_{Pl}^2) . These new corrections cannot be ignored when the black hole mass is close to the Planck mass. However, the corrections are substantially trivial, one could observe this as a consequence of quantum inspection at the level of semi-classical quantum gravity. Note that we have eliminated the terms proportional to $(\alpha L_{Pl}^2 \sqrt{\theta})$ and also $(\alpha L_{Pl}^2 \theta)$ owing to their smallness in nature, to preserve the integrity of Eq. (31), and for one's convenience.

The imaginary part of the action, in high energies, can be written as [41, 42, 43]

$$\text{Im } I = -\frac{1}{2} \Delta S_{NC} = -\frac{1}{2} [S_{NC}(M_I - E) - S_{NC}(M_I)], \quad (32)$$

where S_{NC} is the noncommutative black hole entropy. From this viewpoint the emission rate is proportional to the difference in black hole entropies before and after emission which means that the emission spectrum cannot be accurately thermal at higher energies. From Eq. (31), it is clearly observed that the corresponding tunneling amplitude disagrees with the purely thermal spirit of the spectrum. It can be simply confirmed that the energy conservation or self-gravitational effect plus the additional or combined terms depending on the parameters GUP and noncommutativity (i.e., α and θ , respectively) lead to a Planck-scale statistical correlation function between probabilities of tunneling of two particles with different energies, that is,

$$C(E_1 + E_2; E_1, E_2) = \ln \Gamma(E_1 + E_2) - \ln [\Gamma(E_1)\Gamma(E_2)] \neq 0, \quad (33)$$

where

$$\begin{cases} \Gamma(E_1) = \Lambda \int_0^{M_I - E_1} \Gamma(E_1, E_2) dE_2 \\ \Gamma(E_2) = \Lambda \int_0^{M_I - E_2} \Gamma(E_1, E_2) dE_1 \\ \Gamma(E_1, E_2) = \Gamma(E_1 + E_2) \\ \Lambda = \left[\int_0^{M_I} e^{\Delta S_{NC}} dE \right]^{-1}. \end{cases}$$

This means that the probability of tunneling of two particles with energies E_1 and E_2 is not equal to the probability of tunneling of one particle with their compound energies $E = E_1 + E_2$, as expected from a nonthermal spectrum [8]. Hence subsequent Hawking radiation emissions must be correlated. It is not necessary to write the expression C in terms of energies because it is too long even after simplifying. We have checked our result in some limits. In the limit $\theta \rightarrow 0$, $t = 0$, and $\alpha \neq 0$, one finds

$$\begin{aligned} C(E_1 + E_2; E_1, E_2) = 8\pi E_1 E_2 - 2\pi\alpha L_{Pl}^2 \Big[(16M_I - 15(E_1 + E_2))(E_1^2 E_2 + E_1 E_2^2) \\ - 5(E_1^3 E_2 + E_1 E_2^3) \Big]. \end{aligned} \quad (34)$$

The existence of an additional term depending on the GUP parameter on the right-hand side of Eq. (34) is due to nonthermal GUP correlations. It is evident that in the HUP limit, $\alpha = 0$, with $\theta = 0$ and $t = 0$, we regain the same result as Ref. [8], that is,

$$C(E_1 + E_2; E_1, E_2) = 8\pi E_1 E_2. \quad (35)$$

In the HUP limit with nonzero θ and $t = 0$, we obtain

$$\begin{aligned} C(E_1 + E_2; E_1, E_2) = & 4\pi \left(\left[(M_I - E_1 - E_2)^2 - \frac{3}{2}\theta \right] \mathcal{E} \left(\frac{M_I - E_1 - E_2}{\sqrt{\theta}} \right) \right. \\ & - \left[(M_I - E_1)^2 - \frac{3}{2}\theta \right] \mathcal{E} \left(\frac{M_I - E_1}{\sqrt{\theta}} \right) - \left[(M_I - E_2)^2 - \frac{3}{2}\theta \right] \mathcal{E} \left(\frac{M_I - E_2}{\sqrt{\theta}} \right) \\ & + \left[M_I^2 - \frac{3}{2}\theta \right] \mathcal{E} \left(\frac{M_I}{\sqrt{\theta}} \right) \Bigg) + 12\sqrt{\pi\theta} \left((M_I - E_1 - E_2) e^{-\frac{(M_I - E_1 - E_2)^2}{\theta}} - (M_I - E_1) \right. \\ & \times e^{-\frac{(M_I - E_1)^2}{\theta}} - (M_I - E_2) e^{-\frac{(M_I - E_2)^2}{\theta}} + M_I e^{-\frac{M_I^2}{\theta}} \Bigg). \end{aligned} \quad (36)$$

If one takes the following approximations:

$$\left\{ \begin{array}{l} \mathcal{E} \left(\frac{2(M_I - E_1 - E_2) - t}{2\sqrt{\theta}} \right) \simeq \mathcal{E} \left(\frac{2(M_I - E_1) - t}{2\sqrt{\theta}} \right) \simeq \mathcal{E} \left(\frac{2(M_I - E_2) - t}{2\sqrt{\theta}} \right) \simeq \mathcal{E} \left(\frac{2M_I - t}{2\sqrt{\theta}} \right) \\ e^{-\frac{(2(M_I - E_1 - E_2) - t)^2}{4\theta}} \simeq e^{-\frac{(2(M_I - E_1) - t)^2}{4\theta}} \simeq e^{-\frac{(2(M_I - E_2) - t)^2}{4\theta}} \simeq e^{-\frac{(2M_I - t)^2}{4\theta}}, \end{array} \right.$$

then for $\alpha = 0$, we have

$$C(E_1 + E_2; E_1, E_2) = 8\pi E_1 E_2 \left(1 + \frac{t^2}{2\theta} \right) \mathcal{E} \left(\frac{2M_I - t}{2\sqrt{\theta}} \right). \quad (37)$$

Substituting $t = 0$ into Eq. (37), we get

$$C(E_1 + E_2; E_1, E_2) = 8\pi E_1 E_2 \mathcal{E} \left(\frac{M_I}{\sqrt{\theta}} \right), \quad (38)$$

which is directly obtained from Eq. (36) by applying the approximations given earlier with $t = 0$. For the commutative case, $\frac{M_I}{\sqrt{\theta}} \rightarrow \infty$, the Gauss error function in Eq. (38) tends to unity and one recovers a similar result to Ref. [8]. So, the emission rates for different modes of radiation during evaporation are mutually related to one another from a statistical viewpoint. Moreover, the inclusion of the effects of quantum gravity as a GUP expression plus noncommutativity influences causes the creation of Planck-scale correlations between the different modes of radiation.

3 Summary

In the framework of a noncommutative model of CCS, we have considered a Schwarzschild-like metric for Vaidya solution instead of standard Eddington-Finkelstein metric to observe an exact $(t-r)$ dependent case of the metric. In this situation, we have shown that incorporation of quantum gravity effects, such as GUP, combined with the noncommutativity influences leads to Planck-scale correlations between emitted particles. These features reflect the fact that the information emanates from the black hole as Planck-scale non-thermal correlations within the Hawking radiation and this can shed more light on the information problem in black hole evaporation.

References

- [1] S. W. Hawking, Comm. Math. Phys. **43**, 199 (1975).
- [2] J. Preskill, [arXiv:hep-th/9209058]; D. N. Page, Phys. Rev. Lett. **71**, 3743 (1993); C. R. Stephens, G. 't Hooft and B. F. Whiting, Class. Quant. Grav. **11**, 621 (1994); A. Strominger, [arXiv:hep-th/9501071]; T. Banks, Nucl. Phys. (Proc. Suppl.) **41**, 21 (1995); J. G. Russo, [arXiv:hep-th/0501132] and references therein.
- [3] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
- [4] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).
- [5] S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, Class. Quant. Grav. **19**, 2671 (2002); and Mod. Phys. Lett. A **16**, 571 (2001).
- [6] M. K. Parikh, [arXiv:hep-th/0402166].
- [7] M. Arzano, A. Medved and E. Vagenas, JHEP **0509**, 037 (2005); A. J. M. Medved and E. Vagenas, Mod. Phys. Lett. A **20**, 1723 (2005); and Mod. Phys. Lett. A **20**, 2449 (2005).
- [8] B. Zhang, Q. Cai, L. You and M. Zhan, Phys. Lett. B **675**, 98 (2009); and Annals Phys. **326**, 350 (2011).
- [9] D. Singleton, E. C. Vagenas, T. Zhu and J. Ren, JHEP **1008**, 089 (2010).

- [10] A. Smailagic and E. Spallucci, Phys. Rev. D **65**, 107701 (2002); J. Phys. A **35**, L363 (2002); J. Phys. A **36**, L467 (2003); J. Phys. A **36**, L517 (2003); and J. Phys. A **37**, 7169 (2004).
- [11] P. Nicolini, A. Smailagic and E. Spallucci, Phys. Lett. B **632**, 547 (2006); S. Ansoldi, P. Nicolini, A. Smailagic and E. Spallucci, Phys. Lett. B **645**, 261 (2007); P. Nicolini, Int. J. Mod. Phys. A **24**, 1229 (2009).
- [12] T. G. Rizzo, JHEP **0609**, 021 (2006); R. Casadio and P. Nicolini, JHEP **0811**, 072 (2008); K. Nozari and S. H. Mehdipour, Class. Quant. Grav. **25**, 175015 (2008); S. H. Mehdipour, Commun. Theor. Phys. **52**, 865 (2009); E. Spallucci, A. Smailagic and P. Nicolini, Phys. Lett. B **670**, 449 (2009); K. Nozari and S. H. Mehdipour, Commun. Theor. Phys. **53**, 503 (2010); R. Banerjee, S. Gangopadhyay and S. K. Modak, Phys. Lett. B **686**, 181 (2010).
- [13] B. Zhang, Q. Cai, L. You and M. Zhan, Europhys. Lett. **94**, 20002 (2011).
- [14] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B **216**, 41 (1989); K. Konishi, G. Paffuti and P. Provero, Phys. Lett. B **234**, 276 (1990); M. Maggiore, Phys. Rev. D **49**, 5182 (1994); Phys. Lett. B **304**, 65 (1993); and Phys. Lett. B **319**, 83 (1993); L. J. Garay, Int. J. Mod. Phys. A **10**, 145 (1995); A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D **52**, 1108 (1995); H. Hinrichsen and A. Kempf, J. Math. Phys. **37**, 2121 (1996); C. Castro, Found. Phys. Lett. **10**, 273 (1997); F. Scardigli, Phys. Lett. B **452**, 39 (1999); S. Capozziello, G. Lambiase and G. Scarpetta, Int. J. Theor. Phys. **39**, 15 (2000).
- [15] There is a large body of literature on this subject see for example, R. J. Adler, P. Chen, and D. I. Santiago, Gen. Rel. Grav. **33**, 2101 (2001); A. J. M. Medved and E. C. Vagenas, Phys. Rev. D **70**, 124021 (2004); M. Cavaglia and S. Das, Class. Quant. Grav. **21**, 4511 (2004); B. Bolen and M. Cavaglia, Gen. Rel. Grav. **37**, 1255 (2005); K. Nozari and S. H. Mehdipour, Int. J. Mod. Phys. A **21**, 4979 (2006); and Elec. J. Theor. Phys. **3**, 151 (2006); K. Nozari and A. S. Sefiedgar, Phys. Lett. B **635**, 156 (2006).
- [16] K. Nozari and S. H. Mehdipour, Europhys. Lett. **84**, 20008 (2008); S. H. Mehdipour, Int. J. Mod. Phys. A **24**, 5669 (2009).
- [17] K. Nozari and S. H. Mehdipour, JHEP **03**, 061 (2009).

- [18] R. Fazeli, S. H. Mehdipour and S. Sayyadzad, *Acta Phys. Polon. B* **41**, 2365 (2010).
- [19] M. Nirouei, S. H. Mehdipour and S. Sayyadzad, *Acta Phys. Polon. B* **42**, 1181 (2011).
- [20] S. H. Mehdipour, *Commun. Theor. Phys.* **54**, 845 (2010).
- [21] S. H. Mehdipour, *Phys. Rev. D* **81**, 124049 (2010).
- [22] P. C. Vaidya, *Proc. Indian Acad. Sci. A* **33**, 264 (1951).
- [23] R. W. Lindquist, R. A. Schwartz and C. W. Misner, *Phys. Rev.* **137**, 1364 (1965).
- [24] A. N. St. J. Farley and P. D. D' Eath, *Gen. Rel. Grav.* **38**, 425 (2006).
- [25] H. M. Siahaan and Triyanta, *Int. J. Mod. Phys. A* **25**, 145 (2010); and [arXiv:0811.1132].
- [26] P. S. Joshi, [arXiv:gr-qc/9702036]; A. Papapetrou, *in A Random Walk in general relativity*, edited by N. Dadhich *et al.* (Wiley Eastern, New Delhi, 1985), p. 184.
- [27] C. M. Claudela, K. S. Virbhadra and G. F. R. Ellis, *J. Math. Phys.* **42**, 818 (2001).
- [28] K. S. Virbhadra, D. Narasimha and S. M. Chitre, *Astron. Astrophys.* **337**, 1 (1998).
- [29] K. S. Virbhadra and G. F. R. Ellis, *Phys. Rev. D* **62**, 084003 (2000).
- [30] K. S. Virbhadra, *Phys. Rev. D* **79**, 083004 (2009).
- [31] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, (Freeman, San Francisco) (1973).
- [32] H. Stephani *et al.*, *Exact Solutions to Einstein's Field Equations*, 2nd. ed, (Cambridge University Press, Cambridge) (2003).
- [33] P. Painlevé, *Compt. Rend. Acad. Sci. (Paris)* **173**, 677 (1921).
- [34] E. T. Akhmedov, V. Akhmedova and D. Singleton, *Phys. Lett. B* **642**, 124 (2006); E. T. Akhmedov, V. Akhmedova, T. Pilling and D. Singleton, *Int. J. Mod. Phys. A* **22**, 1705 (2007).
- [35] B. D. Chowdhury, *Pramana* **70**, 593 (2008).

- [36] T. Pilling, Phys. Lett. B **660**, 402 (2008).
- [37] V. Akhmedova, T. Pilling, A. de Gill and D. Singleton, Phys. Lett. B **666**, 269 (2008);
E. T. Akhmedov, T. Pilling and D. Singleton, Int. J. Mod. Phys. D **17**, 2453 (2008).
- [38] P. Kraus and F. Wilczek, Nucl. Phys. B **433**, 403 (1995); and Mod. Phys. Lett. A **9**,
3713 (1994).
- [39] H. Ohanian and R. Ruffini, *Gravitation and Spacetime*, 2nd ed., p. 481 (W. W.
Norton, 1994).
- [40] G. Amelino-Camelia, M. Arzano, Y. Ling and G. Mandanici, Class. Quant. Grav.
23, 2585 (2006).
- [41] E. Keski-Vakkuri and P. Kraus, Nucl. Phys. B **491**, 249 (1997).
- [42] R. Banerjee, B. R. Majhi and S. Samanta, Phys. Rev. D **77**, 124035 (2008); R.
Banerjee and B. R. Majhi, Phys. Lett. B **662**, 62 (2008); and JHEP **06**, 095 (2008).
- [43] S. Massar and R. Parentani, Nucl. Phys. B **575**, 333 (2000).